Modeling and identification of systems

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1 Continuous time systems

1.1 Controller canonical form

$$G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \tag{1.1}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-2} & a_{n-1} \end{bmatrix} \tag{1.2}$$

$$B = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix}^\top \tag{1.3}$$

$$C = \begin{bmatrix} b_0 - b_n a_0 & b_1 - b_n a_1 & \dots & b_{n-1} - b_n a_{n-1} \end{bmatrix} \tag{1.4}$$

$$D = \begin{bmatrix} b_0 \end{bmatrix} \tag{1.5}$$

1.2 Observer canonical form

$$G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \tag{1.6}$$

$$A = \begin{bmatrix} -a_{n-1} & 1 & 0 & \dots & 0 & 0 \\ -a_{n-2} & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_1 & 0 & 0 & \dots & 0 & 1 \\ -a_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{1.7}$$

$$B = \begin{bmatrix} b_{n-1} - b_n a_{n-1} \\ b_{n-2} - b_n a_{n-1} \\ \vdots \\ b_0 - b_n a_0 \end{bmatrix} \tag{1.8}$$

$$C = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{1.9}$$

$$D = \begin{bmatrix} b_0 \end{bmatrix} \tag{1.10}$$

1.3 Leverrier's algorithm

$$(sI - A)^{-1} = \frac{s^{n-1}P_1 + s^{n-2}P_2 + \cdots + P_n}{s^n + a_1 s^{n-1} + \cdots + a_n} \tag{1.11}$$

$$P_1 = I,\ a_1 = -tr(AP_1)/1 \tag{1.12}$$

$$\vdots$$

$$P_n = AP_{n-1} + a_{n-1}I,\ a_n = -tr(AP_n)/n \tag{1.13}$$

$$\text{To verify, } AP_n + a_n I = 0 \tag{1.14}$$

1.4 State space equation from linear differential equation

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_n u^{(n)} + \cdots + b_0 \tag{1.15}$$

$$x_1 = y - h_0 u \tag{1.16}$$

$$x_2 = \dot{x}_1 - h_1 u = \dot{y} - h_0\dot{u} - h_1 u \tag{1.17}$$

$$\vdots$$

$$x_n = \dot{x}_{n-1} - h_n u = y^{(n-1)} - h_0 u^{(n-1)} - \cdots - h_{n-1}u \tag{1.18}$$

$$\dot{x}_n = y^{(n)} - h_0 u^{(n)} - \cdots - h_{n-1}u^{(1)} - h_n u \tag{1.19}$$

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a_{n-1} & 1 & 0 & \dots & 0 \\ a_{n-2} & a_{n-1} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_0 & a_1 & a_2 & \dots & 1 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_n \end{bmatrix} = \begin{bmatrix} b_n \\ b_{n-1} \\ \vdots \\ b_0 \end{bmatrix} \tag{1.20}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-2} & -a_{n-1} \end{bmatrix} \tag{1.21}$$

$$B = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n-1} \\ h_n \end{bmatrix} \tag{1.22}$$

$$C = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \tag{1.23}$$

$$D = \begin{bmatrix} h_0 \end{bmatrix} \tag{1.24}$$

Another method is to take laplace transform and get the controller canonical form

2 Matrix exponential

2.1 Time invariant

Use Cayley-Hamilton method or $\mathcal{L}^{-1}(sI - A)^{-1}$

2.2 Time variant

$$e^{A(t)} = I + \int_{t_0}^{t} A(t)dt + \int_{t_0}^{t_1} A(t_1) \left(\int_{t_0}^{t_2} A(t_2)dt_2 \right) dt_1 + \dots \tag{2.1}$$

3 Discrete time systems

3.1 Solution of difference equation

$$x(k) = \mathcal{Z}^{-1}[(zI - G)^{-1}z]x(0) + \mathcal{Z}^{-1}[(zI - G)^{-1}HU(z)] \tag{3.1}$$

$$G^k = \mathcal{Z}^{-1}[(zI - G)^{-1}z] \tag{3.2}$$

3.2 Conversion of continuous system to discrete system

Let T be sampling period.

$$G = e^{AT} \tag{3.3}$$

$$H = \int_0^T e^{A\lambda} B \, d\lambda \tag{3.4}$$

4 Lagrangian modeling

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} + \frac{\partial D}{\partial \dot{q}} = \text{External force} \tag{4.1}$$