Calculus

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1 Line integrals

If f(x,y) is a scalar

$$\int_C f(x,y)ds = \int_a^b f(h(t),g(t))\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}dt \tag{1}$$

If F(x,y,z)=$P\hat{i} + Q\hat{j} + R\hat{k}$ is a vector

$$\int_C \vec{F}.\vec{dr} = \int_C \vec{F}(r(t)).\vec{r'}(t)dt = \int_C Pdx + Qdy + Rdz \tag{2}$$

Equation (1) is independent of direction of curve whereas equation (2) is dependent on direction of curve taken for integration.

2 Surface integrals

Area of a parameterized surface

$$A = \iint\limits_D \|r_u \times r_v\| dA \tag{3}$$

2.1 Surface integral of a scalar field

Surface integral

$$\iint\limits_S f(x,y,z)dS = \iint\limits_D f(\vec{r}(u,v))\|r_u \times r_v\| dA \tag{4}$$

For $\vec{r}(u,v) = x\hat{i} + y\hat{j} + g(x,y)\hat{k}$, we have

$$\|r_u \times r_v\| = \sqrt{1 + \left(\frac{dg}{dx}\right)^2 + \left(\frac{dg}{dy}\right)^2} \tag{5}$$

2.2 Surface integral of a vector field

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n} dS \tag{6}$$

$$\vec{n} = \frac{r_u \times r_v}{\|r_u \times r_v\|} \tag{7}$$

$$dS = \|r_u \times r_v\| dA \tag{8}$$

$$\Rightarrow \iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot (r_u \times r_v) dA \tag{9}$$

3 Volume integrals

$$\iiint_B f(x, y, z) dV = \iiint_D f(x, y, z) dx\, dy\, dz \tag{10}$$

4 Stoke's theorem

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S} \tag{11}$$

To check the orientation of the surface, use gradient information.

5 Gauss' theorem or Divergence theorem

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_V \vec{\nabla} \cdot \vec{F} dV \tag{12}$$

6 Green's theorem

2D version of Stoke's theorem. Because $d\vec{S}$ in a plane points in z direction, we only have to compute the corresponding component in the curl of function.

$$\int_C P\, dx + Q\, dy = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy} \right) dA \tag{13}$$

7 Fourier series

$f(t)$ is periodic with period $T_0 = \frac{1}{f_0}$
Version 1

$$f_{T_0}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_0 t n};\ where\ f_0 = \frac{1}{T_0}\ and\ c_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} f_{T_0} e^{j2\pi f_0 t} \tag{14}$$

Version 2

$$f(t) = \sum_{n=0}^{\infty} a_n cos(nf_0t) + b_n sin(nf_0t) \tag{15}$$

$$a_n = \frac{1}{2T} \int_0^T f(t)cos(nf_0t)\, dt \tag{16}$$

$$b_n = \frac{1}{2T} \int_0^T f(t)sin(nf_0t)\, dt \tag{17}$$